



PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2018

Question/Answer booklet

# MATHEMATICS SPECIALIST UNIT 1

Section One:  
Calculator-free

Your Name

Your Teacher's Name

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet

### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Question	Mark
1		5	
2		6	
3		7	
4			

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	94	65
<b>Total</b>					100

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**Section One: Calculator-free****(52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

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**Question 1****(9 marks)**

- (a) Hilbert numbers are defined as numbers that are one greater than the multiples of four, such as 5, 9, 13, 17, 21, 25..... Prove that the product of two Hilbert numbers is a Hilbert number.

**(4 marks)**

- (b) Prove that if a square and a circle have the same perimeter  $P$ , the circle will have the greater area.

**(5 marks)**

## Question 2

(10 marks)

Given that  $\mathbf{a} = -2\mathbf{i} + 8\mathbf{j}$  and  $\mathbf{b} = 5\mathbf{i} - 6\mathbf{j}$ , find:

(a)  $|\mathbf{a} + \mathbf{b}|$ . (3 marks)

(b) the unit vector parallel to  $\mathbf{a} + \mathbf{b}$ . (1 mark)

(c) a vector that is parallel to  $\mathbf{a} + \mathbf{b}$  but with a magnitude of 5. (1 mark)

(d)  $a$  in terms of  $p$  and  $q$  where  $p = 2i + j$  and  $q = -3i + 3j$ . (5 marks)

## Question 3

(4 marks)

Use *Proof by Contradiction* to show that there is no positive integers  $a$  and  $b$ , such that

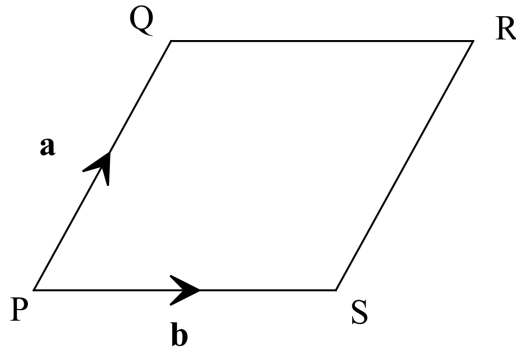
$$a^2 - 4b^2 = 1$$

**Question 4**

**(6 marks)**

$PQRS$  is a rhombus.  $PQ = a$  and  $PS = b$ .

(a) Find  $PR$  in terms of  $a$  and  $b$ . (1 mark)



(b) Find  $QS$  in terms of  $a$  and  $b$ . (1 marks)

(c) Hence, use a vector method to show that the diagonals of a rhombus are perpendicular to each other. (4 marks)

**Question 5****(7 marks)**

(a) Prove the recurring decimal  $3.\overline{210}$  is rational by expressing it as a fraction. (3 marks)

(b) Prove irrationality by contradiction for  $\sqrt{10}$ . (4 marks)



**Question 6****(8 marks)**

(a) Show that the second number in row  $n$  of Pascal's triangle is always  $n$  for  $n > 0$ . (2 mark)

(b) Prove that the third number of any row of Pascal's triangle is always  $\frac{1}{2} n (n - 1)$ . (2 mark)

(c) Show that for any row in Pascals' Triangle, the  $(r + 1)$ th number is  $\frac{n-r}{r+1}$  times the  $r$ th number, that is,  ${}^n C_{r+1} = \frac{n-r}{r+1} \times {}^n C_r$  assuming that  ${}^n C_r = 1$  and  $n > 0$ . (4 marks)

## Question 7

(8 marks)

- (a) Prove that any 3-digit number is divisible by three if the sum of its digits is divisible by three.  
[Hint: let the units digit be  $a$ , the tens digit be  $b$ , and the hundreds digit be  $c$ .]

(3 marks)

- (b) Use mathematical induction to prove that  $2^{6n} + 3^{2n-2}$  is always divisible by 5, for  $n \in \mathbb{N}$ .  
(5 marks)

**Additional working space**

Question number: \_\_\_\_\_

**Additional working space**

Question number: \_\_\_\_\_

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Question number: \_\_\_\_\_

**Acknowledgements**